**Soil Mechanics II and Exercises [Midterm Exam]**

May 29, 2024 (Wed.) 8:45-10:15 Kyotsu 1 lecture room

Attention:

- The exam consists of three questions for which you are provided with three answer sheets. Write down your name and ID number on every answer sheet. Use one answer sheet per question and answer them in sequence, starting from [Question 1]. If the front page of an answer sheet is insufficient to complete your answer, use **the back page of the same answer sheet** after clearly indicating your intent.
- Scores for each question are equally weighted.
- In addition to personal writing instruments, use of non-programmable calculators and rulers are permitted. However, programmable calculators and calculator functions of mobile phones are prohibited.
- Wherever necessary, specify the units in your answers.
- Any attempt at cheating on the exam will result in failed credit of the course and serious penalties.

[Question 1]

(1) A saturated clay layer of thickness $H$ lies above an impermeable rock layer. Assume that the groundwater level is at the ground surface. After the rapid application of a load over the clay layer, the consolidation process is considered.

(1-1) Terzaghi's consolidation equation is given below.

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2}$$

Here, $u$ is the excess pore water pressure, $c_v$ is the coefficient of consolidation, $t$ is time, and $z$ is the coordinate along the vertical direction. Derive Terzaghi's consolidation equation by presenting the four conditional equations required for the derivation. For the derivation, use the parameters shown in Table 1.

Table 1

| | | | | | |
|---|---|---|---|---|---|
| $v$ | : Flow velocity of pore water | $k$ | : Hydraulic conductivity | $\gamma_w$ | : Unit weight of water |
| $u$ | : Excess pore water pressure | $\varepsilon$ | : Volumetric strain | $m_v$ | : Coefficient of volume change |
| $\sigma'$ | : Effective stress | $\sigma$ | : Total stress | $c_v$ | : Coefficient of consolidation |
| $t$ | : Time | $z$ | : Vertical coordinate | | |

(1-2) Write down the boundary condition for the upper surface of the clay layer (ground surface, $z = 0$).

(1-3) Write down the boundary condition for the lower surface of the clay layer (in contact with the impermeable layer, $z = H$).

(1-4) Assuming that the initial pore water pressure $u_0$ is constant regardless of the depth, the solution of the consolidation equation is given by the following equation. Let the horizontal axis represent the excess pore water pressure $u$, and the vertical one be the depth $z$. Draw schematic diagrams of excess pore water pressure distribution at the initial time, during consolidation and at the end of consolidation, respectively.

$$u(z,t) = \sum_{n=0}^{\infty} \frac{2u_0}{M} \sin\frac{M}{H} z \cdot \exp(-M^2 T_v)$$

where, $M = {}^{(2n+1)\pi}/_{2}$, $T_v = {}^{c_v t}/_{H^2}$.

(2) Fig. 1 shows a pressure bulb when a concentrated load $P$ acts on the ground surface. Answer the following questions.

(2-1) Explain the pressure bulb. Also, what is a possible use for a pressure bulb?

(2-2) When a concentrated load $P$ acts on the ground surface, the vertical stress increment $\Delta\sigma_z$ at any point $A(x, y, z)$ is expressed as follows,

$$\Delta\sigma_z = \frac{3Pz^3}{2\pi r^5}$$

where, $r^2 = x^2 + y^2 + z^2$ .

Express the shape of the pressure bulb due to the concentrated load $P$ acting on the ground surface in terms of the polar equation when using the polar coordinates $(r, \theta)$ shown in Fig. 1.

(Hint: Express $r$ as a function of $r_0$ and $\theta$)

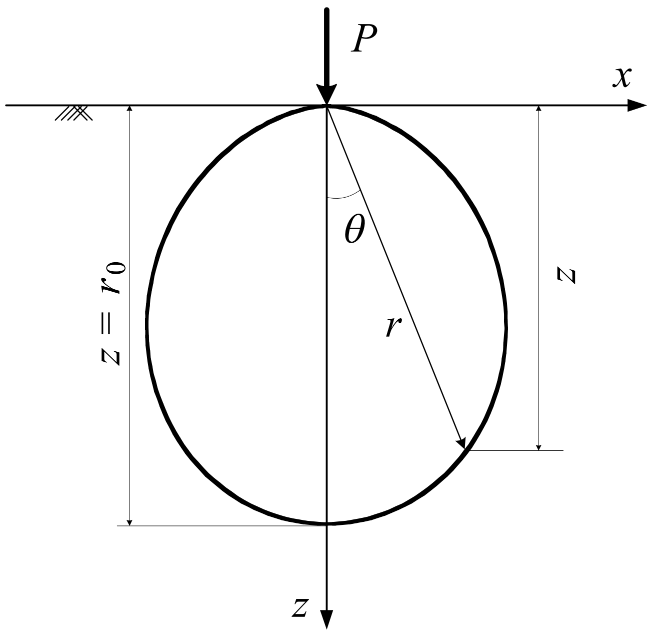


Fig. 1

[Question 2]

Using the following procedure, a triaxial compression test was performed on a fully saturated clay specimen with a preconsolidation stress of 100 kPa. First, the specimen was isotropically consolidated at a confining pressure of 200 kPa. After consolidation, the confining pressure was increased by 50 kPa under undrained condition, and the pore pressure coefficient, $B$, was measured. Then, the confining pressure was returned to 200 kPa, and undrained shear test was conducted at constant confining pressure while excess pore pressure was measured. The saturated clay specimen had a cohesion c' = 0, and an internal friction angle $\phi' = 30°$, and a pore pressure coefficient at failure $A_f = 0.9$.
Answer the following questions.

(1) Is the clay specimen in the normal consolidation or overconsolidation state when isotropically consolidated at a confining pressure of 200 kPa?

(2) Find the pore water pressure ($\Delta u$) and $B$ value when the confining pressure is increased by 50 kPa.

(3) Find the deviatoric stress ($\sigma_{1f} - \sigma_{3f}$) at failure.

(4) Find the excess pore water pressure ($\Delta u_f$) at failure.

(5) Find the maximum effective principal stress ($\sigma'_{1f}$) and minimum effective principal stress ($\sigma'_{3f}$) at failure.

(6) Draw Mohr's effective stress circle at failure and the Mohr-Coulomb's failure envelope. Also, using the polar method, indicate the "pole" in the figure and draw the failure surface.

(7) Find the effective normal stress $\sigma'_f$ and shear stress $\tau_f$ acting on the failure surface of the specimen.

(8) Draw the effective stress path during the undrained shear test after consolidation is completed, show the failure envelope in the $p'$-$q$ plane where the average effective stress ($p'$) is represented by the $x$-axis and the deviatoric stress ($q$) is represented by the $y$-axis. Also, find ($p'$, $q$) when the failure envelope is reached.

## [Question 3]

Consider a retaining wall of height $H$ (internal friction angle $\phi'$, cohesion $c'$, unit weight $\gamma$ for the backfill soil) as shown in Fig. 2. The cohesion $c'$ is assumed to be zero. Friction between the wall surface and the backfill soil is assumed to be negligible. The groundwater table is sufficiently deeper than the bottom of the retaining wall. Answer the following questions.

(1) For the following two cases, give the name of the earth pressure acting on the retaining wall.

(a) the retaining wall is displaced in the direction to the left ($\delta_-$) in the figure

(b) the retaining wall is displaced in the direction to the right ($\delta_+$) in the figure

(2) The earth pressure acting on the retaining wall is determined using the Coulomb theory. An assumed failure surface is shown by the dashed line in the figure. The weight of the soil wedge shown in the figure (behind the wall and above the assumed failure surface) is $W$, the normal force acting perpendicular to the assumed failure surface is $N$, the frictional force acting parallel with the assumed failure surface is $T$. The resultant force of earth pressure acting on the retaining wall is $P$. For the two cases (a) and (b) in (1), show the force polygon diagram acting on the soil wedge and express the resultant force $P$ using $W$, $\theta$ and $\phi'$.

(3) For case (b) of (1), determine the resultant force $P$ using differentiation with respect to $\theta$.

(4) The earth pressure acting on the retaining wall is determined using the Rankine theory. For the two cases (a) and (b) in (1), show Mohr's circles at failure and determine the earth pressures acting on the wall at a depth $z$ from the surface of the backfilled soil.

(5) For case (b) of (1), determine the resultant force of earth pressure, $P$, by integration and check that it agrees with (3).

(6) The ground conditions of the backfill soil are again checked by experiment and it is found that the cohesion $c'$ is not zero. For case (a) in (1), find the resultant force $P$ acting on the wall at a depth $z$ from the surface of the backfilled soil.

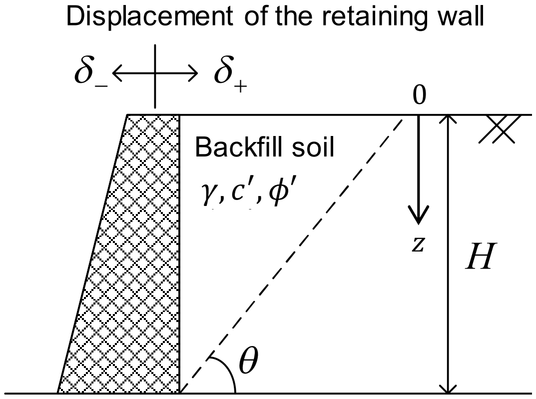


Fig. 2